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Spectral measurements of turbulent momentum transfer in fully developed pipe flow

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Measurements of the spectral components of turbulent momentum transfer for fully developed pipe flow are presented. The results indicate that near the wall ($y^+ < 15$) two types of momentum transfer processes occur. A net positive transfer takes place in the higher frequency range of the energy-containing part of the turbulence spectrum whereas a net negative transfer returns low momentum to the wall region at the lower end of the spectrum. Examination of the turbulence at various y^+ shows that the significant features of the turbulence spectra scale on frequency at any given Reynolds number, thus leading to an interpretation of the flow structure which is consistent with the hydrogen-bubble visualization data of Runstadler, Kline & Reynolds (1963). The results are consistent with a flow model in which disturbances extend from the sublayer to the core of the flow. Recent turbulent heat transfer measurements are also interpreted successfully by this model.

1. Introduction

Considerable progress in the visualization of turbulent flows has been made in the last few years. As a result several distinct structural features of such flows have been identified. From the work of Runstadler *et al.* (1963), Kim, Kline & Reynolds (1971) and Grass (1971) it is possible to identify at least the following basic flow pattern in bounded turbulent flow. In a small region very close to the wall, turbulence appears to be produced by a relatively regular eruption of low momentum fluid which propagates or diffuses through a considerable thickness of the whole boundary layer. A distinct structure is detectable near the boundary (Runstadler *et al.* 1963; Morrison, Bullock & Kronauer 1971) but as the distance from the wall increases, this structure becomes more difficult to identify as it blends into the remaining general turbulent field. An important implication of this work is that the boundary layer has a continuous structure rather than one consisting of several different layers where each layer displays its own characteristic behaviour. From this it can be expected that turbulence measurements at different distances from the wall should exhibit common features which reflect this continuity in structure.

Many spectral measurements of hot-wire signals have been made in the past in an attempt to distinguish periodic structural features of turbulent boundary layers. Such work has led to considerable knowledge of spectral properties of the longitudinal, normal and transverse fluctuating velocity in the outer layers of flows but owing to technical difficulties such as sensor size and signal processing, little information is available concerning the spectral properties of the normal fluctuating velocity and in particular the turbulent momentum transfer very close to a solid boundary. Recent examples of such measurements include the work of Bradshaw (1967) in an equilibrium and a strongly retarded boundary layer down to $y^+ = 218$ and of Lawn (1971) in fully developed pipe flow to $y^+ = 109$. From visualization studies it is clear, however, that the effect of the wall structure is small at such large non-dimensional (y^+) distances from the wall.

In the present study spectral turbulent momentum transfer results have been obtained at small y^+ values in order to determine certain features of the wall structure. These measurements have been made possible by the development of a new hot-wire anemometer technique for the measurement of the normal velocity fluctuation.

A second aspect of turbulent shear flows is the scaling of spectral information in view of the visualization data and the study of burst rates and their scaling on outer flow variables as found by Narahari Rao, Narasimha & Badri Narayanan (1971). In the present work it is found that, at a given Reynolds number, the most appropriate frequency scaling is in terms of a parameter which is directly proportional to frequency.

2. Apparatus and experimental techniques

The working section of the wind tunnel used for the present results consisted of a 5·34 in. inside diameter ($a = 2{\cdot}67$ in.), 30 ft long extruded aluminium tube in which traverses were performed at 63 tube diameters from the entry. Air was supplied to the tube from a centrifugal fan through a settling chamber designed to eliminate swirl and large-scale turbulence. The entry from this chamber to the working section consisted of a bellmouth entry followed by a screen and boundary-layer trip in order to promote flow development.

Some of the measurements reported were performed with a conventional X-meter hot-wire anemometer. The wires were 0·0004 in. in diameter, 0·055 in. long and of platinum–rhodium–ruthenium alloy (Sigmond–Cohen type 851). Each wire formed part of a complete, fully compensated, constant-current, hot-wire circuit. Signal separation was performed as described by Bullock & Bremhorst (1969). The frequency response of the combined hot-wire, amplification and signal-processing equipment was flat to within 2 % from 4 Hz to 2 kHz, the lower limit being set by a simple RC filter with a 2 s time constant used to remove any d.c. levels at the input to the analog-computer signal-processing circuits. With this X-meter it was possible to explore the flow to the edge of the buffer layer without gross distortion of the results due to wire-length and separation effects.

In order to overcome the resolution problem and permit measurements closer to the wall than is possible with an X-meter Walker & Bullock (1972) developed a new technique for the measurement of the normal velocity component. The method relies on sensing the instantaneous temperature at a point in the wake close behind a conventional hot wire. For distances downstream from the hot wire less than the smallest scale of the turbulence, the temperature wake is dominated by molecular diffusion, thus giving a Gaussian temperature distribution normal to the velocity vector. The wake temperature at a point, and hence a point on this Gaussian curve, is detected with a fully compensated, constant-current, hot-wire anemometer operated at very low overheat (cold wire) so that it acts as a resistance thermometer. Using appropriate calibration techniques and analog computer processing of the signals from the hot and cold wires then yields the instantaneous flow direction and hence the longitudinal and normal velocity fluctuations, from which the turbulent momentum transfer $\langle uv \rangle$† can be obtained by analog multiplication and averaging.

Since the two wires are very close together (0·003 in.) and are parallel to the flow boundary, the spatial resolution is almost identical to that of a single hot wire and measurements down to $y^+ \simeq 4$ are possible in quite small diameter working sections without limiting the flow Reynolds number to very small values as is necessary for most visualization studies. Measurements of Reynolds stress in fully developed pipe flow with this method even at $y^+ = 6$ agreed well with calculated values as shown in §3. The signals proportional to u and v were recorded on an FM tape recorder and played back for subsequent analysis.

Spectral analyses in the case of the X-wire results were performed using Bruel & Kjaer type 2107 constant-percentage bandwidth analysers with the noise bandwidth set at 10·5 % of the centre frequency, whereas those taken with the new technique were processed through analog filters with a 20 % bandwidth. The effect of the different filters on the final result was found to be of no consequence. For spectral cross-correlations, signal channels were carefully aligned, so that no errors due to phase could influence the results. All signal processing including time averaging was performed on an EAI 231R analog computer, thus achieving the long averaging time required to minimize statistical variability of individual readings.

3. Mean flow and turbulence measurements

Friction factor measurements agreed with the Blasius relationship to within 3 % for the Reynolds number range of 10^4–2×10^5. Mean velocity profiles also agreed well with the universal velocity profile. A comparison of measured longitudinal and normal turbulence components with those published in the literature is shown in figure 1 for $Re_{0a} = 26780$ (Re_{0a} is based on the centre-line velocity and pipe radius). The turbulent momentum transfer as measured with the new technique is compared in figure 2 with that calculated from the mean velocity distribution. Measured and previously published cross-correlation coefficients between the longitudinal and normal velocity fluctuations are compared in figure 3. Laufer's lower Reynolds number data (open circles) can be disregarded for this comparison as scatter is considerable and the trends are inconsistent with the higher Reynolds number data. One of the present author's (K.B.) X-wire measurements at the higher Reynolds number are slightly in error because of wire-length effects, which have been shown to reduce the spectral cross-correlation coefficient (Bremhorst 1972).

† The symbol $\langle\ \rangle$ is used throughout to denote long-time averages.

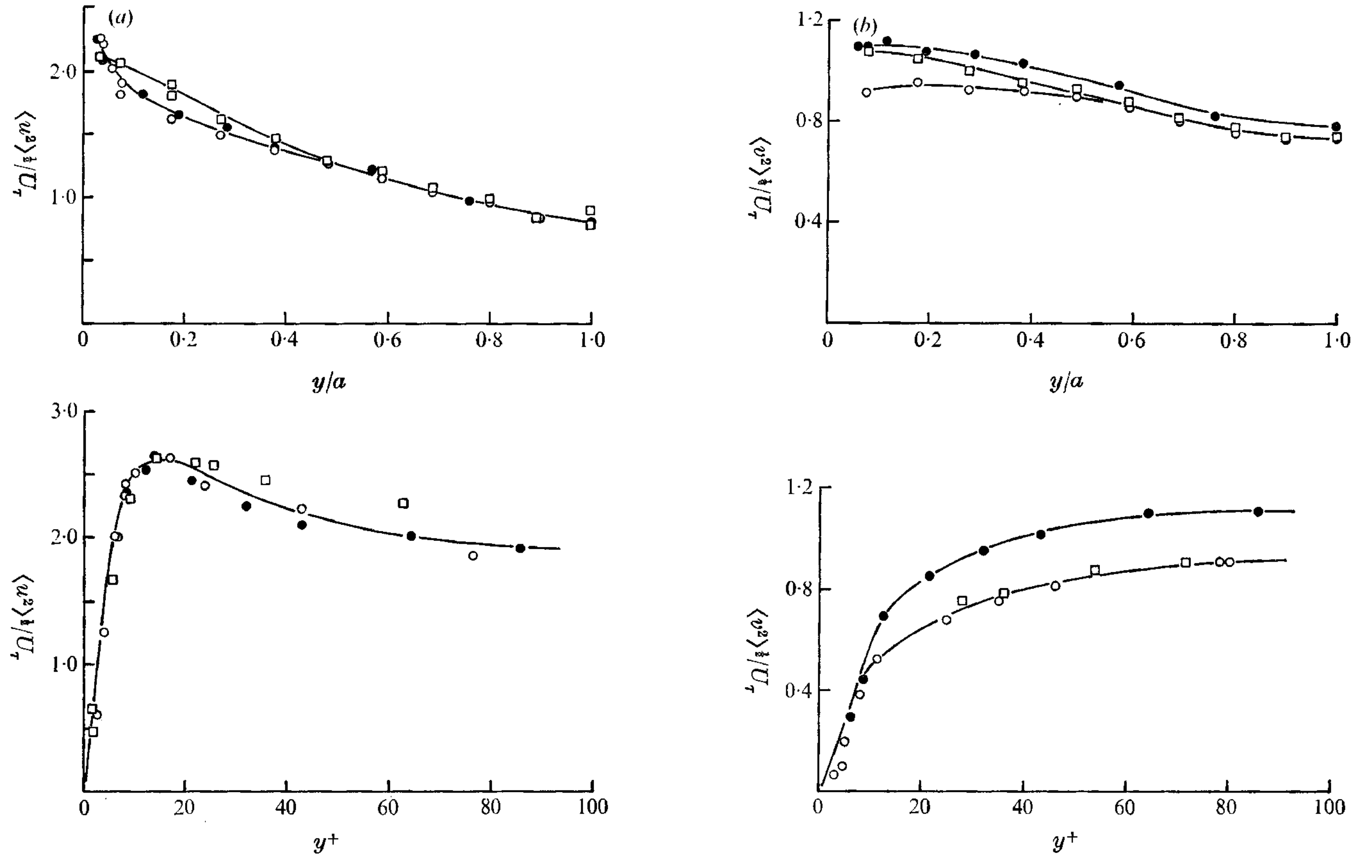

FIGURE 1. Comparison of measured and published turbulence intensities. $y^+ = 100$ corresponds to $y/a = 0{\cdot}088$. (*a*) Longitudinal turbulence intensity. (*b*) Normal turbulence intensity.

	$Re_{0a} = U_0 a/\nu$	U_τ (ft/s)	
●	26 780	0·865	present authors
○	25 000	0·42	Laufer (1954)
□	250 000	3·49	Laufer (1954)

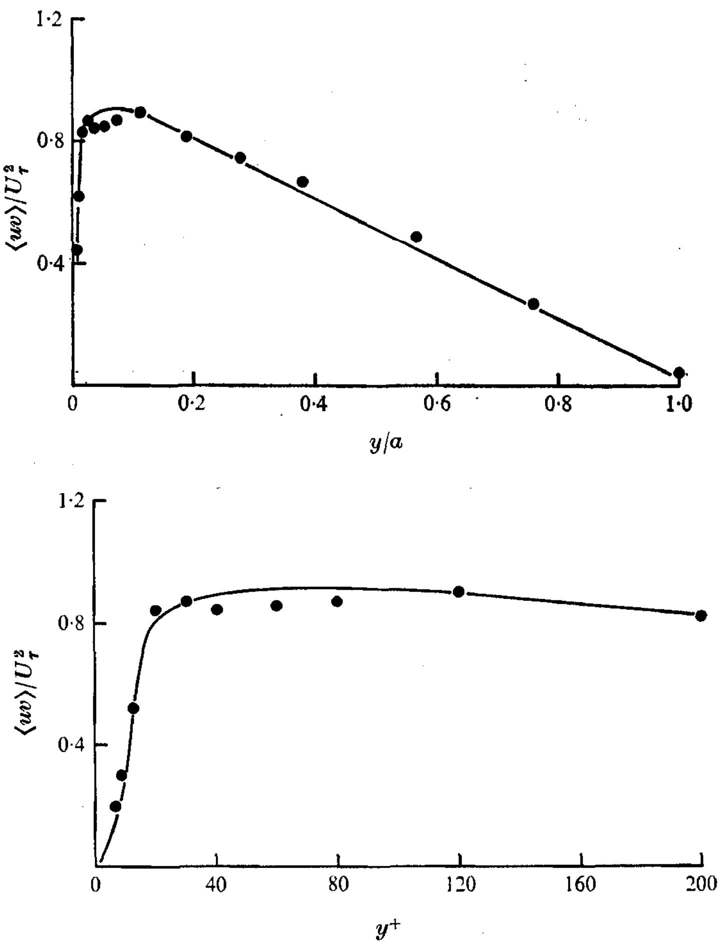

FIGURE 2. Comparison of measured and calculated turbulent momentum transfer: $Re_{0a} = U_0 a/\nu = 26\,780$, $U_\tau = 0{\cdot}865$ ft/s. ——, calculated from measured mean velocity profile. $y^+ = 200$ corresponds to $y/a = 0{\cdot}176$.

4. Spectral cross-correlation coefficients $R_{uv}(f)$

Spectral cross-correlation coefficients for two different Reynolds numbers obtained with a conventional X-wire probe in fully developed smooth-tube flow are shown in figure 4, where the longitudinal velocity fluctuation u is measured in the stream direction and the normal velocity fluctuation v in the radial direction; that is, towards the wall. The spectral cross-correlation coefficient is defined as

$$R_{uv}(f) = \langle u(f)\, v(f) \rangle / [\langle u^2(f) \rangle \langle v^2(f) \rangle]^{\frac{1}{2}}.$$

The results show remarkably similar trends for the different Reynolds numbers. At the pipe centre-line the correlation coefficient at all frequencies was found to be $0 \pm 0{\cdot}04$, thus giving an indication of the quality of signal separation when obtaining the u and v velocity components.

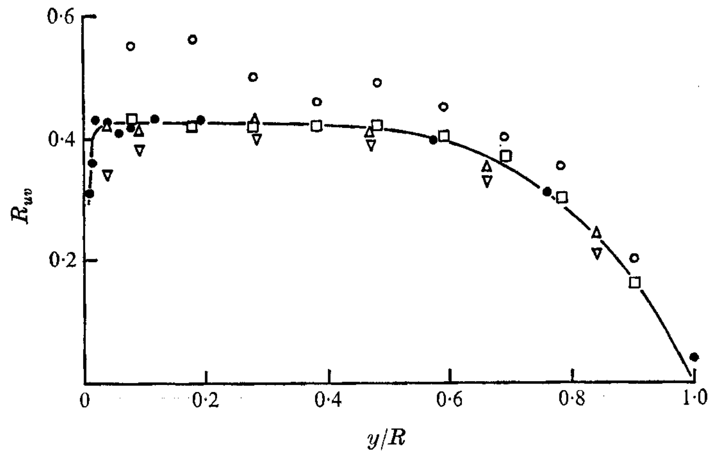

FIGURE 3. Comparison of measured and previously published cross-correlation coefficients.

	$Re_{0a} = U_0 a/\nu$	U_τ (ft/s)	
●	26 780	0·865	present authors
○	25 000	0·42	Laufer (1954)
□	250 000	3·49	Laufer (1954)
△	36 100	1·065	Bremhorst (1969)
▽	95 000	2·58	Bremhorst (1969)

For the majority of the flow, but still well removed from the wall region, the low frequency flow components show the highest correlation between u and v. Thus very effective mixing between fluid layers of different momentum is achieved by these flow components whereas the correlation of the higher frequency components shows a steady decrease, hence indicating a trend towards local isotropy. With the aid of the u and v spectral results shown later (figure 6) it is seen, however, that this trend towards isotropy is evidenced only well outside the range of energy-containing frequencies.

The most significant feature of the results is the reversal of the above trends as the wall is approached. The correlation of the low frequency components declines rapidly towards zero in the wall region whereas the correlation of the high frequency components which approach isotropy in the core shows a marked increase. A more organized high frequency flow pattern must therefore exist in the wall region. It also implies that near the wall only a limited frequency band plays a dominant role in the momentum transfer process, as is confirmed by the cross-spectral results in §6. Obviously a most interesting region is, therefore, that below $y^+ = 50$.

Similar measurements using the technique of Walker & Bullock (1972) outlined earlier are presented in figure 5. Although the results were obtained at a different Reynolds number, the trends are identical to those found in figure 4; in addition, it is seen that the correlation coefficient of the low frequency components shows a reversal in sign close to the wall. These components must, therefore, produce a reversal in the momentum exchange near the wall such that low momentum fluid is returned to the wall or high momentum is ejected from the wall or both phenomena occur simultaneously.

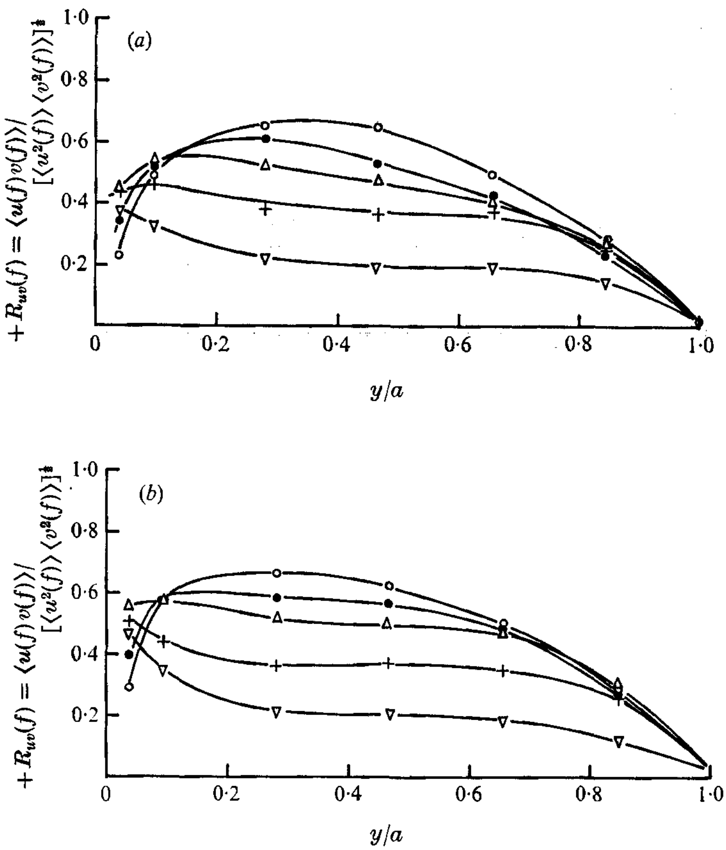

FIGURE 4. Spectral cross-correlation coefficients obtained with a conventional X-wire probe. U_0 = centre-line velocity. (*a*) $Re_{0a} = U_0 a/\nu = 36\,100$, $U_\tau = 1{\cdot}065$ ft/s. (*b*) $Re_{0a} = U_0 a/\nu = 95\,000$, $U_\tau = 2{\cdot}58$ ft/s; Bremhorst (1969).

		Frequency, f (Hz)	
	$2\pi f/U_0$ (ft^{-1})	(*a*)	(*b*)
○	1·17	5	13·2
●	3·60	15	40·7
△	7·04	30	79·6
+	16·0	150	407·0
▽	117·0	500	1320

Lawn's (1971) data show identical trends but the negative correlations are not in evidence as the measurements reported did not extend below $y^+ = 109$. Thus, although the results presented so far are at different Reynolds numbers, and were collected by different workers and different methods, all trends are completely consistent and hence the effects found must exist physically and require an explanation.

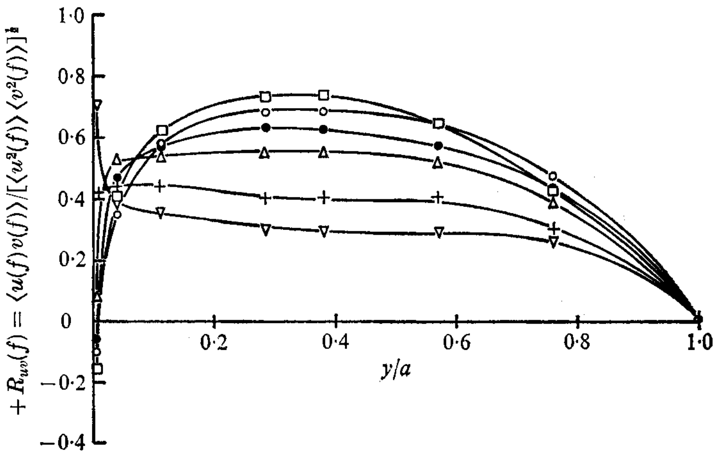

FIGURE 5. Spectral cross-correlation coefficients close to the wall using the Walker–Bullock (1972) method; $Re_{0a} = U_0a/\nu = 26780$, $U_\tau = 0{\cdot}865$ ft/s, U_0 = centre-line velocity

	Frequency, f (Hz)	$2\pi f/U_0$ (ft^{-1})
□	1·5	0·46
○	3·82	1·17
●	11·7	3·60
△	22·9	7·04
+	117	36·0
▽	382	117

5. Basic flow structure assumptions and method of spectral data presentation

For the presentation of power and cross-spectra the less popular linear–log plot was chosen to show the energy-containing frequency bands directly. In addition, this method of plotting accentuates the transition from the empirically established energy-containing range where $\langle u^2(f)\rangle$ varies approximately as f^{-1} to the universal equilibrium range where $\langle u^2(f)\rangle$ varies approximately as $f^{-\frac{5}{3}}$. Since the aim was to identify a basic flow phenomenon associated with the ejection of some disturbance from the wall region into the outer layers, frequency rather than wavenumber scaling was used. This was because the frequency of the ejections at the wall and the frequency of the basic disturbance as detected in some distorted fashion further out in the flow must be the same and, therefore, an abscissa directly proportional to frequency will show the energy change of the disturbance with distance from the wall directly. The argument implies that, if a disturbance originates near the wall, it will be stretched in the direction of the shear as it progresses through the shear layers so that it enlarges in extent but its basic frequency will be identical at all distances from the wall.

This argument is consistent with the results of Narahari Rao *et al.* (1971), who found that bursts scale on free-stream or core-flow variables, thus indicating that the frequency associated with the bursts is constant across the flow.

Linear–log spectral plots of energy versus frequency are presented so that

$$\int_0^\infty f\frac{\langle u^2(f)\rangle}{U_\tau^2}\,d\ln f = \langle u^2\rangle/U_\tau^2,$$

where $\langle u^2(f)\rangle$ represents the energy of $\overline{u^2}$ associated with the frequency f; similarly for $\langle v^2(f)\rangle$ and $\langle u(f)\,v(f)\rangle$.

6. Spectra of u, v and uv

The spectral results of figure 6 are plotted using U_τ as the normalizing factor rather than the local mean-square turbulence so that energy levels can be compared directly at different distances from the wall.

The plots of the longitudinal velocity fluctuation energy spectrum indicate that in the log region of the flow this energy is centred about two frequency regions (8–30 and 80–200 Hz). (The emphasis is on the energy-containing range of frequencies. No attempt is made to attach importance to the peaks themselves but refer also to discussion in § 7.)

It is also seen that the bulk of the normal fluctuating velocity energy occurs at significantly higher frequencies than that of u, and that the total energy of the normal velocity fluctuations at the lower frequencies is almost negligible compared with that of the longitudinal velocity fluctuation. Undoubtedly, this high degree of anisotropy is a result of the restraining influence of the wall. At the higher frequencies (in this case over 500 Hz) the universal equilibrium range is approached with a consequent trend towards local isotropy.

The spectral turbulent momentum transfer results of figure 6 (*c*) show several new and quite distinct features. Near the wall, $y^+ \simeq 8$–12, a dominating net positive transfer of momentum occurs at high frequencies in a relatively narrow band whereas at low frequencies a small net negative transfer of momentum takes place. Thus the role of the low frequency band changes considerably as the distance from the wall increases. Near the wall the latter produces a return of low momentum fluid to the wall (assuming that the transfer of high momentum from the wall is not significant; refer also to § 7), whereas at distances remote from the wall (even in the conventional buffer layer) it produces the reverse situation. Hence the spatial extent of the phenomenon causing this reversal in behaviour must decrease very rapidly with y.

7. Discussion of results

Emphasis in the interpretation of the present results will be placed on the u spectrum. Strictly speaking, the spectrum of the total fluctuating energy $\overline{q^2}$ should be considered rather than that of the resolved components if it is intended to trace a disturbance through the flow since a simple reorientation of a disturbance with respect to the co-ordinate axes can have a significant effect on the observed results for individual components. A summation of the u, v and w spectra is therefore required, but unfortunately extensive data concerning the w spectra, especially in the wall region, are not available. The spectral results of

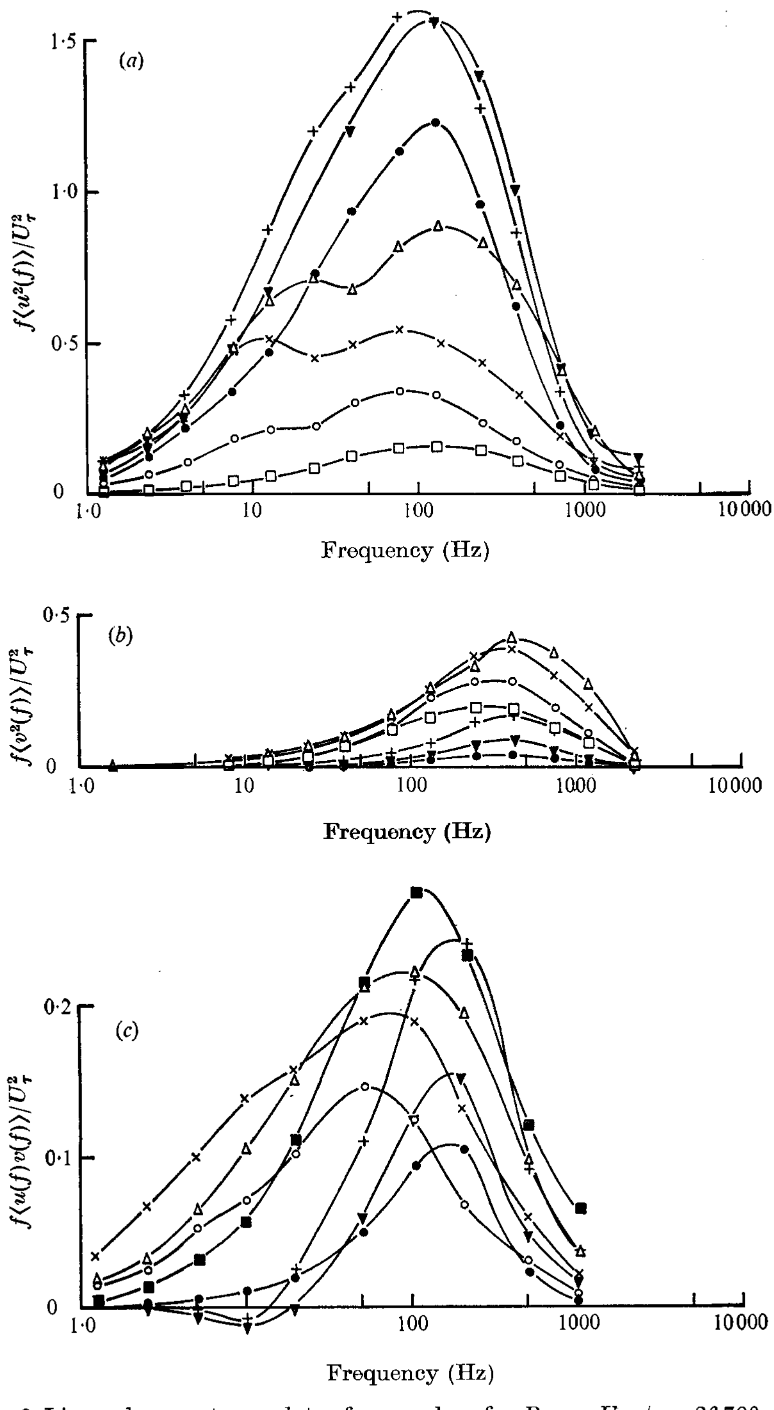

FIGURE 6. Linear–log spectrum plots of u, v and uv for $Re_{0a} = U_0 a/\nu = 26780$, $U_\tau = 0{\cdot}86$ ft/s. (*a*) Spectra of longitudinal turbulence intensity. (*b*) Spectra of normal turbulence intensity. (*c*) Cross-spectra of turbulent momentum transfer. y^+: ●, 6·46; ▼, 8·6; +, 12·9 ■, 21·5; △, 64·6; ×, 215; ○, 646; □, 1131 (= centre-line).

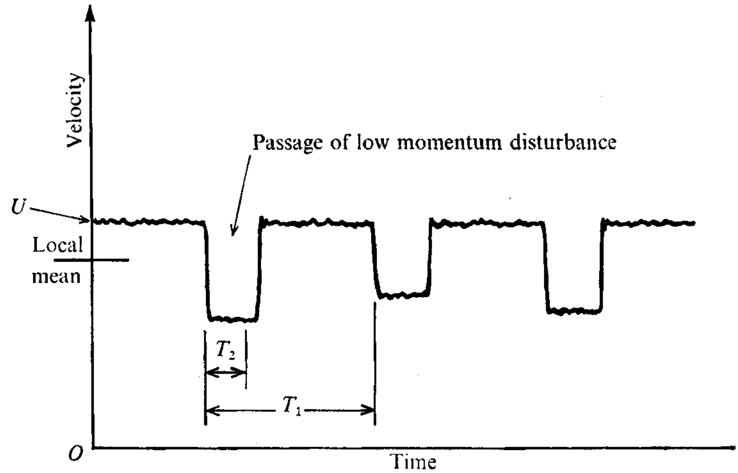

FIGURE 7. Simplified time history of turbulence at a point near the wall.

Lawn (1971) indicate that the spectra of q will differ little in shape from those of u since both v and w are of a much lower magnitude. In fact, the net effects of the latter components is to align the higher frequency peaks seen in the u spectra so that these peaks occur at practically the same frequency at all distances from the wall. Transition to the universal part of the spectrum is, therefore, seen to take place in the same frequency range at all points in the flow, even where the flow is penetrated by an organized structural feature.

Thus, if the flow is viewed as a generally uncorrelated turbulent field superimposed on which is a group of semi-organized disturbances, the following picture emerges. Disturbances are generated near the wall with a frequency (or time scale) defined by the lower band of the u spectra. These are then ejected away from the wall. Associated with each disturbance is another frequency range (or time scale), representing its signature, and this gives rise to the upper frequency band if the time scales of the two processes are well separated. A very simplified time history (Eulerian picture) of such a process is shown in figure 7, where T_1 is the time between disturbances, T_2 is the time associated with the background turbulence and also that within a burst, U is the undisturbed instantaneous velocity at the point and it is assumed that $T_2 \ll T_1$ so that little or no interaction occurs between disturbances. A spectral analysis of such a signal (remembering that both T_1 and T_2 have large variations associated with them) would produce a spectrum with two distinct bands, the lower frequency one representing the process associated with T_1 and the higher frequency one that associated with T_2. T_2 is shown associated with a burst because it is assumed that this is comparatively directly related to the frequency range of the background turbulence, which presumably is produced from the oscillations observed by Kim *et al.* (1971) during the second stage of the bursting cycle. Also, T_2 must be less than the duration of the burst since it is only a part of the whole bursting cycle.

Assuming then that a disturbance originating at the wall propagates through the flow it is expected that that frequency $1/T_1$ would be observed everywhere and the energy associated with T_2 could decrease as dissipation and breakup of the basic disturbance occurs owing to shearing actions. This is precisely what is observed in the results of figure 6. The fact that the energy in the upper frequency band initially drops more rapidly with distance from the wall than that in the lower band indicates that the more energetic frequency components (as distinct from the energy-containing range of frequencies) which occur in the lower frequency band persist further through the flow than the less energetic ones in the higher frequency band. This interpretation and the fact that all spectra align on a frequency basis give strong support to the view that disturbances originating at the wall propagate through the flow and thus characterize it. The signature associated with T_2 would, of course, depend on whether the disturbance is a longitudinal vortex, transverse vortex or a completely different type of structure the exact nature of which is of no real consequence in the current arguments.

Although the above results are not entirely conclusive in themselves since interpretation of limited spectral data can only be speculative, the visual data of Runstadler *et al.* (1963) and Kim *et al.* (1971) and findings of Grass (1971) using conditional sampling complement the above view considerably. Perhaps, the best evidence would be spectral cross-correlations of longitudinal velocity components spatially separated in the normal direction with one wire at $y^+ = 5$–10 and the other traversing across the flow. For such measurements to be consistent with the above view, high correlations would have to be obtained across the flow for large normal separations in the low frequency band and significantly lower correlations would have to be obtained in the high frequency band, due allowance having been made for the direction of propagation of the disturbance relative to the wall. Unfortunately, no such data are available to date.

Before proceeding to a discussion of the spectral momentum transfer results, it is necessary to verify how T_1 can be identified with the bursting period T_B of Kim *et al.* (1971). Since no universal scaling between boundary-layer and pipe spectra could be devised, autocorrelation functions of hot-wire longitudinal velocity fluctuations were measured using a PAR Model 100 cross-correlator. From the results the frequency corresponding to the time to the first re-rise of the correlation function was measured and found to fall within the lower frequency band of the spectral data as identified above. Since this method of frequency identification, although only approximate, is identical with that used by Kim *et al.*, a direct comparison between the two sets of results is obtained.

The most significant results are contained in the cross-spectra of turbulent momentum transfer as already noted in §6. At low frequencies (associated with T_1) in the wall region a negligible and even a net negative contribution to the turbulent momentum transfer was observed and contrasts sharply with the large positive contributions observed further from the wall. This may be due to several phenomena. Since the present results are only single-point measurements, time-averaged over all events, no firm conclusions can be reached, but the visualization studies of Kim *et al.* (1971) clearly show that, in the region affected, frequent movements towards the wall of low speed streaks occur associated with the breakup of a burst. The time scale of this return of low momentum fluid must, therefore, be of the order of T_1 thus giving a negative contribution at the frequency associated with T_1. No similar observations as part of the bursting cycle have so far been reported with respect to high momentum fluid moving away from the wall and this is therefore assumed to play only a minor role. Significant negative turbulent momentum contributions would also be consistent with the large values of the 'fraction of turbulence production' parameter measured by Kim *et al.* (1971) at $y^+ \simeq 6$–15, which agrees well with the y^+ range of 6·5–13 over which the present measurements indicate the existence of such contributions.

The flow process now suggested is that near the wall ($y^+ \simeq 6$–13) the positive momentum transfer at low frequencies already exists as part of the basic pattern of ejection of disturbances. Thus it is simply the spectral contribution associated with the time T_1 of figure 7. The fact that it is not detected in the time-averaged results is due to the overriding contribution of the low momentum inflow to the wall. As the distance from the wall increases, this wall component decreases rapidly and the only remaining low frequency momentum transfer is then positive as shown by the results.

8. Relevance of flow model to heat transfer

The flow structure interpretation used also explains the spectral correlations between the longitudinal velocity and the stream temperature fluctuations in fully developed pipe flow reported by Bremhorst & Bullock (1970). They showed that the low frequency components of the longitudinal velocity and stream temperature fluctuations are almost perfectly correlated even in the core of the flow whereas the higher frequency components show a significantly lower correlation everywhere except very close to the wall. This is to be expected if the flow consists of periodic bursts which propagate right from the wall to the core of the flow. Because disturbances originate at the heated wall they consist of hot, low momentum fluid which passes through the flow, resulting in a fluctuating temperature field identical to that of the velocity field with period T_1 (as in figure 7). The high correlation between momentum and temperature at low frequency right across the flow with the highest correlation occurring at the wall is further evidence that the flow structure is a continuous phenomenon extending from the wall to the centre of the flow. On the other hand, the low and high momentum fluid within a disturbance will be at approximately the same temperature. This leads to a lower net correlation between momentum and temperature at frequencies associated with T_2.

9. Conclusions

A newly developed hot-wire anemometer technique for the measurement of turbulent momentum transfer very close to a solid boundary has revealed the existence of a flow component which returns low momentum fluid to the wall. Results further from the wall indicate that the low frequency component of the wall structure most probably consists of two parts: one which ejects low momentum fluid into the bulk of the flow and an overriding one which returns low momentum fluid to the wall. The major net transfer of low momentum fluid from the wall takes place in a relatively narrow band in the high frequency range near the wall, but further out in the flow it takes place over the full energy-containing range of frequencies.

Frequency scaling of spectra shows that the transition of turbulence to the universal range occurs in the same frequency band throughout the flow. The present findings are also consistent with turbulent heat transfer results.

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